Equilibrium Values in Energy Markets

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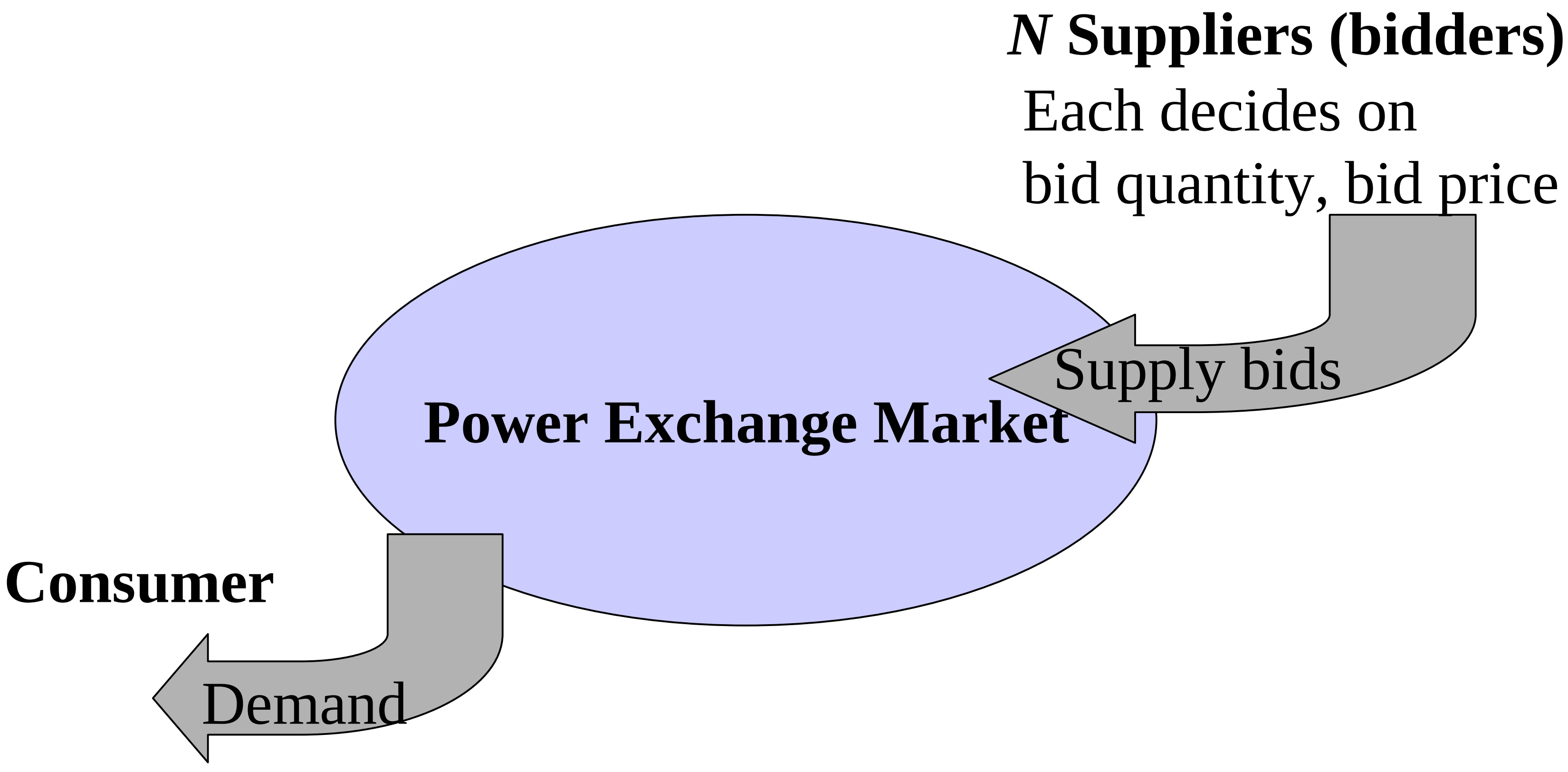
Problem Overview

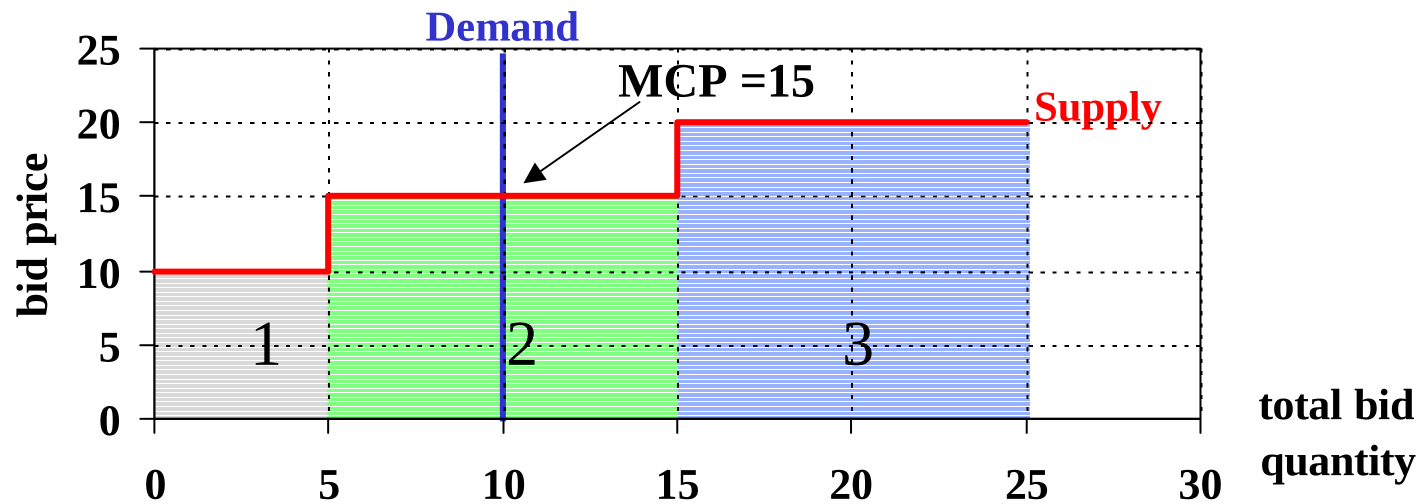


Problem Overview (cont.)

Demand
{d =10}

Supply bids
{(x_i,p_i), i=1,..N}
{(5,10),(10,15),(10,20)}

Market Clearing Process



Model

- Bidder i has unit cost c_i
- Single bid per bidder : (x_i, p_i)
  - x_i = bid quantity of bidder i
  - p_i = bid price of bidder i (assume discrete)

$$p_i \in \{l\varepsilon \mid l = 0,1,\ldots,O\} \forall i$$

  - $\mathbf{b} = [(x_1,p_1), \ldots, (x_N,p_N)]$
- Sell at spot: Market clearing price

$$\pi(\mathbf{b}, d) = \left\{ \min_j \; p_j : \sum_{i \in I(j)} x_i \geq d \right\}, I(j) = \{i : p_i \leq p_j\}$$

Model(cont.)

- Non sealed bid : infinite adjustment
- Dispatch quantity of Bidder *i*

$$q_i(\mathbf{b}, d) = \begin{cases} 0 & \text{if } p_i > \pi(\mathbf{b}, d) \\ x_i & \text{if } p_i < \pi(\mathbf{b}, d) \\ \overline{q}_i(\mathbf{b}, d) & \text{if } p_i = \pi(\mathbf{b}, d) \end{cases}$$

- Dispatch of marginal bidders follows order determined from submission time of bids

Model(cont.)

- Bidder i 's payoff

$$f_i(\mathbf{b}) = E_D\left[(\pi(\mathbf{b}, D) - c_i)q_i(\mathbf{b}, D)\right]$$

- Objective: Find the worst Nash equilibrium for consumer: **b*** = $\{(x_i^*, p_i^*), i = 1,\ldots, N\}$ such that

$$f_i\left(\left[(x_1^*, p_1^*),\ldots,(x_i, p_i),\ldots,(x_N^*, p_N^*)\right]\right) \le f_i\left(\left[(x_1^*, p_1^*),\ldots,(x_i^*, p_i^*),\ldots,(x_N^*, p_N^*)\right]\right)$$

for all feasible bids, (x_i, p_i), for bidder i, and all $i = 1,\ldots, N$

and $\pi(\mathbf{b}^*, d)$ is maximized

Worst equilibrium point

- Worst equilibrium MCP = highest possible bid price

$$p_i^* = c_i \quad \text{for } i = 1, \dots, N-1$$

$$\sum_{i=1}^{N-1} x_i^* = d - \varepsilon$$

$$p_N^* = O\varepsilon$$

$$x_N^* \geq \varepsilon$$

Preventing the worst point

- Cannot lower bid quantity (cannot withdraw bids)
- Market stability condition

$$\overline{D} \leq \sum_{\forall i \neq j} x_i \quad \text{for } j = 1, \ldots, N$$

at least one bidder cannot dispatch

Assumptions & Notation

- Assume distinct bidders:

$$\left|c_i - c_j\right| > 2\varepsilon \quad \forall i \neq j$$

- Fixed bid quantity: bidders can adjust only bid price
- Given a number *x*,

$\lfloor x \rfloor = \max\{i\varepsilon \mid i\varepsilon \leq x,\ i=0,..,O\}$ and

$\lceil x \rceil = \min\{i\varepsilon \mid i\varepsilon \geq x,\ i=0,..,O\}$

Results

- For deterministic demand case, the highest possible equilibrium MCP must be in $\{\lfloor c_i \rfloor, i = 1, \ldots, N\}$
- For stochastic demand case, the highest possible equilibrium MCP must be in $\{\lfloor c_i \rfloor, \lceil c_i \rceil, \lceil c_i \rceil + \varepsilon,\ i = 1, \ldots, N\}$
- Algorithm to find highest equilibrium price and bidding strategies

Results (cont.)

- This equilibrium price is not unique
- Special case: $x_i = x$ for all $i = 1, \ldots, N$: Equilibrium point with highest spot price

$$p_i^* = \lfloor c_{i+1} \rfloor \quad \forall i = 1, \ldots, N-1$$

$$\pi = \lfloor c_k \rfloor$$

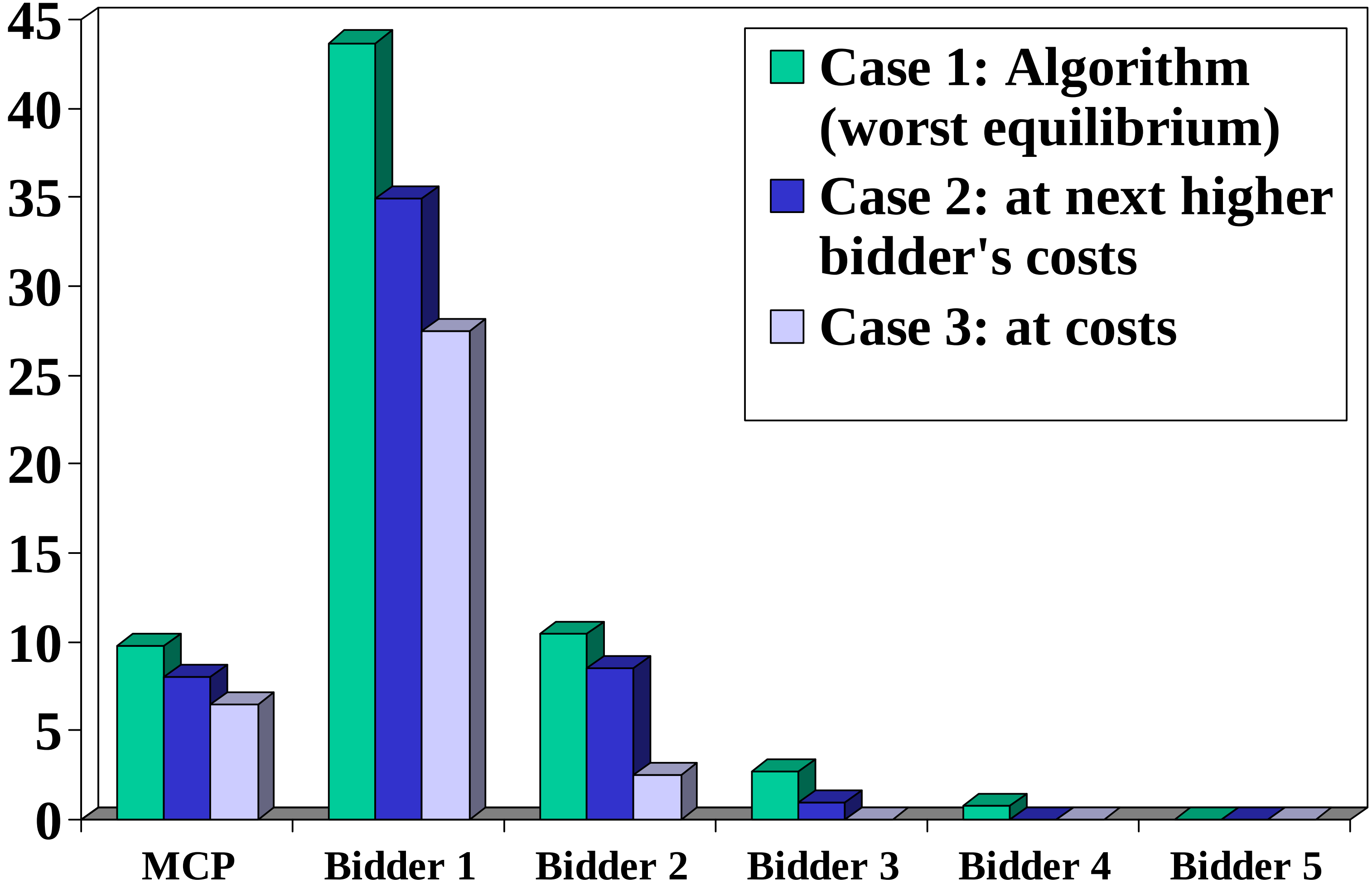
where k is the first undispatched bidder, given all bidders bid at their costs.

Numerical Example

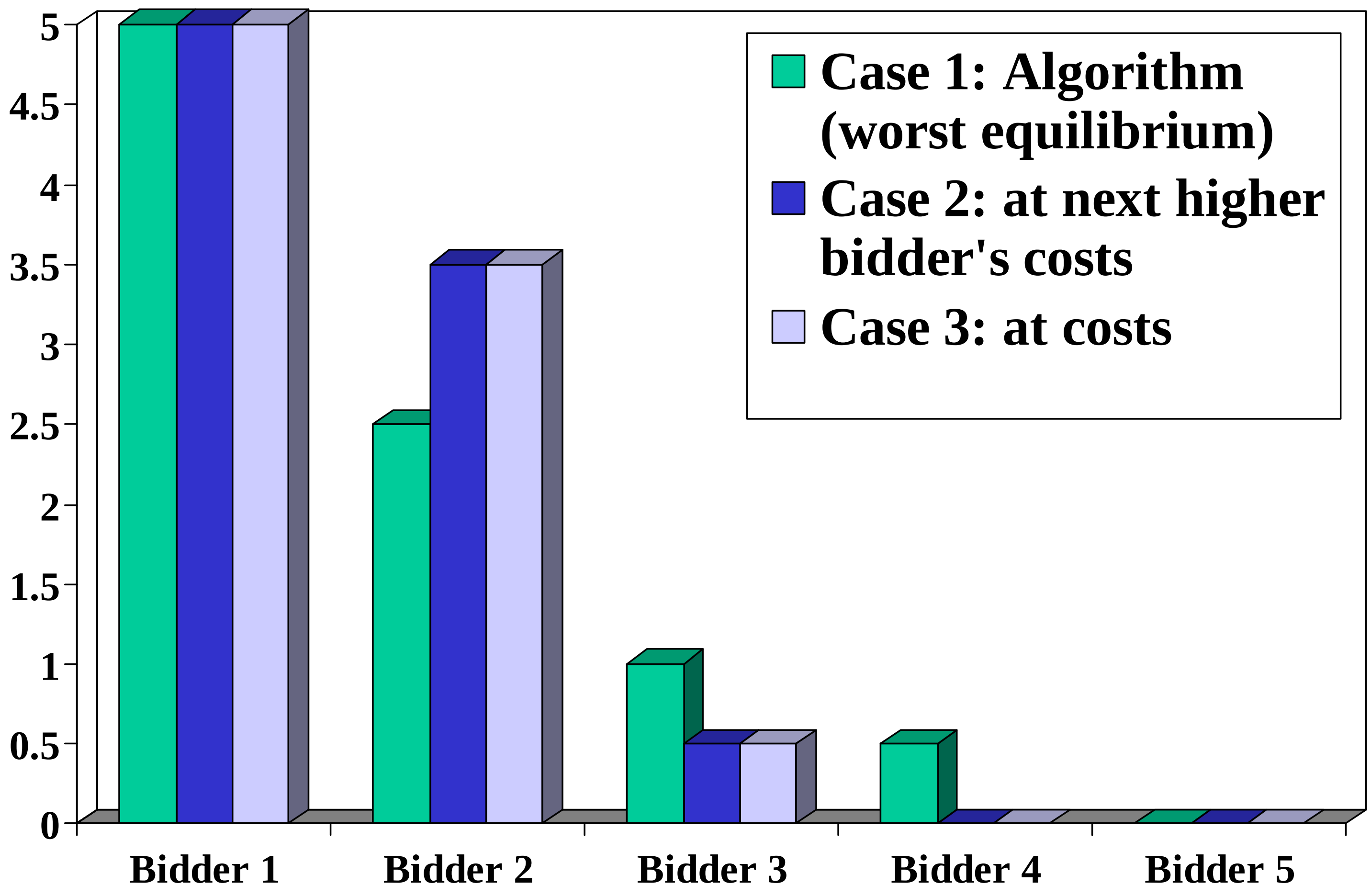
i	C_i	X_i
1	1.01	5
2	6.01	5
3	7.01	1
4	9.01	1
5	10.51	11

- $\varepsilon = 0.01$, demand = 7 and 11 w.p. 0.5

Comparison of MCP and Payoff



Comparison of Dispatch Quantity



Extensions

- Find conditions when this market is efficient, i.e., equilibrium dispatches optimize system as a whole
- Include fixed cost of generation
- Analyze multi-period problem: cost depends on the dispatch of the previous period
- Allow multiple bids per bidder

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